Areas of Strengths and Growth Opportunities for Students Who Achieved Level 2 on the 2022–2023 Grade 9 Assessment of Mathematics

Introduction

This study provides detailed information regarding students' mathematical strengths and growth opportunities on the Grade 9 Assessment of Mathematics, with a focus on students achieving Level 2.

This work expands on EQAO's previously released Student Achievement on Mathematics Curriculum Strands (Education Quality and Accountability Office [EQAO], 2023c) by analyzing student responses to single-selection questions to identify strengths and potential growth opportunities in mathematical understanding. Detailed examples are provided to illustrate student misunderstandings as well as instructional implications and recommendations that educators can use to move student achievement from Level 2 to the provincial standard. Although the analysis provided in this report focuses on students at Level 2, the themes and instructional implications that emerge are useful for all students, as they highlight areas required for strong mathematical understanding.

Method

The data used for this study include responses from all participating students at public school boards in Ontario who received an assessment level outcome from the Grade 9 Assessment of Mathematics in the 2022–2023 school year.

In the Grade 9 assessment, each student is presented with 50 operational and four field-test questions. These questions assess student knowledge and skills across the Number, Algebra, Data, Geometry and Measurement, and Financial Literacy strands.[1] As outlined in the Grade 9 Assessment of Mathematics Framework (EQAO, 2023a), 20% of the 50 operational questions are allocated to the Number strand, 36% to the Algebra strand, 16% to the Data strand, 16% to the Geometry and Measurement strand and 12% to the Financial Literacy strand. Each question is also mapped to one of three of the categories of knowledge and skills: Knowledge and Understanding, Application or Thinking.

The Grade 9 Assessment of Mathematics uses a multi-stage computer adaptive testing model (EQAO, 2020) that adapts to student performance as the student progresses through the assessment sessions. The assessment is composed of different types of selected-response questions such as drag and drop, drop-down menu, ordering, and single- and multiple-selection questions (EQAO, 2023a). For this study, only single-selection questions, commonly known as multiple-choice questions, were included in the analysis from the five assessed strands and three knowledge and skills categories. Field-test questions were not included. This resulted in the selection of 168 operational questions for an item-by-item analysis as outlined below.

The item-by-item analysis was conducted by calculating the percentage of students at each achievement level who selected each answer option for each question. Response patterns were then examined to understand the potential mathematical misunderstandings and misconceptions that led students to select incorrect answers.

Table 1 provides an example of the analysis process for one question, in which the percentage of students who selected each option is calculated for students at each achievement level. In Table 1, 20% of students achieving Level 2 selected option A as opposed to the correct option C. Option A was then analyzed to determine why students might have selected it and what this selection revealed about potential mathematical misunderstandings and misconceptions.

Students achieving below Level 1 were not included in the analysis due to small sample sizes. Responses for students in English- and French-language schools were analyzed separately, and major differences are described in detail in this report where applicable.

Table 1. Percentage of students who chose each option, by achievement level

Level	A	B	C	D	No Response	Total
Level 1	24%	27%	33%	15%	1%	100%
Level 2	20%	13%	59%	7%	1%	100%
Level 3	7%	2%	89%	2%	–	100%
Level 4	1%	–	99%	–	–	100%

Findings

Classification of Mathematical Themes

Through the analysis of item level performance, patterns of students' mathematical understanding were grouped under four themes: Proportional Reasoning, Algebraic Reasoning, Computational Thinking and Mathematical Language. By grouping mathematical understanding into these four areas, student strengths and areas for growth were identified. Each of these themes will be explored in detail in the following sections by using examples from the Resource: Released Questions, Grade 9 Assessment of Mathematics (EQAO, 2023b). As well, instructional implications and recommendations for improving student performance are provided for each theme.

Theme 1. Proportional Reasoning

One of the most prevalent and noticeable themes that appeared during the analysis of student responses was proportional reasoning, which "permeates mathematics and is often considered the foundation to abstract mathematical understanding" (Ontario Ministry of Education, 2012, p. 2). As such, proportional reasoning appears across the Ontario mathematics curriculum. It is defined as "reasoning that is based on equal ratios and that involves an understanding of the multiplicative relationship in the size of one quantity compared with another" (Ontario Ministry of Education, 2020b, p. 567).

When examining the answer choices of Grade 9 students at Level 2 within the context of proportional reasoning, both strengths and areas for improvement became apparent. In terms of strengths, students achieving Level 2 were able to solve simple problems involving proportional reasoning. For example, when the total cost for a specific number of items was given, students could determine the total cost for a different number of items. Students at Level 2 also seemed to understand how to convert measurement units (e.g., litres to millilitres) under familiar contexts and how to convert values to and from scientific notation. For example, students were able to convert values written using scientific notation with positive exponents (e.g., $3.2 \times 10^7 = 32\ 000\ 000$).

Areas for improvement for students achieving Level 2 were apparent in questions involving rates, percentages and ratios; proportionality within dimensions; proportional changes to a budget and representations of linear relationships. The examples below explore each of these findings in detail.

Rates, Percentages and Ratios

Students achieving Level 2 had some difficulty in answering questions that involved solving problems with proportions, specifically those involving ratios. When presented with questions that involved writing a ratio in simplified form or solving more complex equations involving proportions and/or mixed numbers, only about 20% of students at Level 2 answered this type of question correctly as compared to more than 50% of students at Level 3. Example 1, taken from the *Resource: Released Questions, Grade 9 Assessment of Mathematics* (EQAO, 2023b), illustrates these findings and how students struggled with understanding multiplicative relationships between quantities.

Example 1

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 12)

There are a total of 90 red and yellow tiles.

There are 5 times as many red tiles as yellow tiles.

How many red tiles are there?

A 15 tiles

B 18 tiles

C 72 tiles

D 75 tiles

Objective: Students were asked to determine the number of red tiles given the total number of red and yellow tiles and considering that there are five times as many red tiles compared to yellow tiles.

Overall Curriculum Expectation: C1, Algebraic Expressions and Equations. To answer correctly, students need “an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations” (Ontario Ministry of Education, 2022, p. 105).

Mathematical Misunderstanding: When answering this type of question, students achieving Level 2 often selected options that likely indicate they can divide but may not have constructed an equation involving fractions which can be used to arrive at the correct answer. In this example, students at Level 2 could have divided 90 by 5 and selected an answer of 18 tiles (option B) instead of constructing equivalent ratios such as $5/6 = x/90$ and then solving to determine the correct answer of 75 tiles (option D). Students who struggled with questions like Example 1 showed a lack of understanding of the concepts required to solve equations with fractions that involve proportional reasoning.

Proportionality Within Dimensions

Students achieving Level 2 experienced some difficulty in understanding proportions in relation to area and volume. When presented with questions that involved proportional changes to area or volume, only about one third of students at Level 2 answered correctly as compared to two thirds of students at Level 3. Example 2, from the *Resource: Released Questions, Grade 9 Assessment of Mathematics* (EQAO, 2023b), illustrates these findings and how students at Level 2 may have relied on additive thinking instead of using the multiplicative thinking required to solve problems that involve proportionality.

Example 2

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 28)

A frame is in the shape of a rectangle.

A second rectangular frame has triple the length and one third of the width of the first frame.

Complete this statement.

The area of the second frame is ____________ the area of the first frame.

A the same as

B half the size of

C three times the size of

D six times the size of

Objective: Students were asked to proportionally compare the area of two rectangular frames when the second frame has a length that is triple the length of the first frame and a width that is one third the width of the first frame.

Overall Curriculum Expectation: E1, Geometry and Measurement Relationships. To answer correctly, students need an "understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations" (Ontario Ministry of Education, 2022, p. 176).

Mathematical Misunderstanding: When answering this type of question, students achieving Level 2 often selected options that indicated difficulty with spatial proportions such as doubling and tripling when comparing the area of two rectangles. Some students relied on additive thinking (3 + 3) to arrive at option D. Mathematical errors of this nature highlight the importance of the development of proportional reasoning skills and the distinction between additive and multiplicative thinking (Lamon, 2012; Ontario Ministry of Education, 2012).

Proportional Changes to a Budget

Understanding proportional changes to a budget is another area of growth for students at Level 2 on the Grade 9 Assessment of Mathematics. Some students struggled when given a question involving the comparison of fixed percentage increases and how these can proportionally change expenses and income. For example, students could be asked to compare how much more a 5% increase in both an income and an expense (that are not equal) affects their savings from one month to another. Students who struggled with questions of this nature often selected options that represented no change, which may indicate they believed that a 5% change in both was equivalent in value even though the income and expense amounts were different.

Representations of Linear Relationships

Students achieving Level 2 struggled with the proportional aspects inherent in the representation of linear relationships and the calculation of the slope (rate of change). When presented with questions that required the determination of the rate of change based on a table of values or a graphical linear representation, students who had difficulty in this area often selected options that indicated they struggled when the increments of the scales being used on the x- and y-axes were different. This concept is important not only for the interpretation of graphs (data interpretation) but also for an understanding of rates of change. Questions with different scales on the axes do not easily allow for the counting of the change in y and change in x and, hence, the calculation of the rise ($y_2 - y_1$) and run ($x_2 - x_1$). To correctly answer this type of question, students must understand the rate of change as a difference between the start and end values, which requires a more in-depth understanding of rate. These findings are also explored in the following section on algebraic reasoning, as the mathematical understanding overlaps between the different themes.

Implications and Recommendations for Proportional Reasoning

Proportional reasoning appears across the Ontario mathematics curriculum and "is often considered the foundation to abstract mathematical understanding" (Ontario Ministry of Education, 2012, p. 2). This widespread nature is evident in the examples provided above, which detailed student difficulties in correctly answering questions mapped to different strands across the curriculum. A key aspect that appeared in the analysis was the tendency for students who struggled with proportional reasoning to use counting or additive methods for questions that required multiplicative thinking, which "requires thinking about situations in relative rather than absolute terms" (Ontario Ministry of Education, 2012, p. 5). Developing multiplicative thinking is not only important for answering mathematical problems that require multiplication and division, but also has implications for learning in the Algebra, Geometry and Measurement, Data and Financial Literacy strands.

Recommendations

- Make use of multiple assessment types and opportunities to determine which aspects of proportional reasoning students already understand, and what next steps are needed to reach each student's learning goals. This could include the use of released questions from the Grade 9 Assessment of Mathematics that involve proportional reasoning as part of a pre- and post-assessment.
- Allow students to practise with a variety of proportional tasks in multiple contexts across strands that foster connections across mathematical topics. This practice could include providing students with various tasks involving the use of different scales on number lines, on axes on graphs and in tables of values.
- Consider activities with math manipulatives (including virtual manipulatives) that allow students to explore different proportional and non-proportional problems, or consider tasks that involve classifying additive and multiplicative problems followed by classroom discussion (Van Dooren et al., 2010).

Theme 2. Algebraic Reasoning

Algebraic reasoning appeared as an important theme during the analysis of student responses. The Ontario Ministry of Education defines algebraic reasoning as "our ability to notice patterns and generalize from them. Algebra is the language that allows us to express these generalizations in a mathematical way. Formulas, such as the area of a rectangle = length × width ($A = l \times w$), are derived through such generalizations" (Ontario Ministry of Education, 2013, p. 3).

When examining the answer choices of Grade 9 students at Level 2 within the context of algebraic reasoning, both strengths and areas for improvement became apparent.

In terms of strengths, students at Level 2 could identify simple algebraic expressions or equations. For example, when presented with a written representation and asked to select the correct algebraic expression or equation, most students were able to correctly match the written situation to the corresponding expression or equation. Also, students successfully identified an initial value from a graphical representation of a linear relationship. Students were also able to identify the most appropriate lines of best fit from a selection of scatter plots.

Areas for potential growth for students achieving Level 2 were found within the theme of algebraic reasoning. These include use of the distributive property, understanding of formulas and comparison of various representations of relations. The examples below illustrate each of these growth areas.

Distributive Property

Students achieving Level 2 had difficulty simplifying algebraic expressions, especially with the distributive property. When presented with a simplification question such as Example 3, approximately 25% of students at Level 2 answered it correctly as compared to about 70% of students at Level 3. As well, a difference was noted here for French-language students achieving Level 2 as compared to Level 3. Approximately 40% of French-language students at Level 2 answered this type of question correctly as compared to 70% of French-language students at Level 3.

Example 3

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 10)

What is a simplified form of this expression?

$-3x(4x^2 - 5)$

A $-12x^2 - 15x$

B $-12x^3 - 5$

C $-12x^2 + 15$

D $-12x^3 + 15x$

Objective: Students were asked to simplify an expression involving a variable.

Overall Curriculum Expectation: C1, Algebraic Expressions and Equations. To answer correctly, students need an “understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations” (Ontario Ministry of Education, 2022, p. 105).

Mathematical Misunderstanding: When answering this type of question, students achieving Level 2 often selected options that indicate difficulty with use of the distributive property. Instead of distributing $-3x$ to both terms within the brackets, which would have given the correct answer ($-12x^3 + 15x$), students often selected option B ($-12x^3 - 5$), in which $-3x$ is distributed only to the first term and not to the second term. Students who struggled here also struggled with the multiplication of two monomials involving rational numbers in addition to the multiplication of two monomials involving variables.

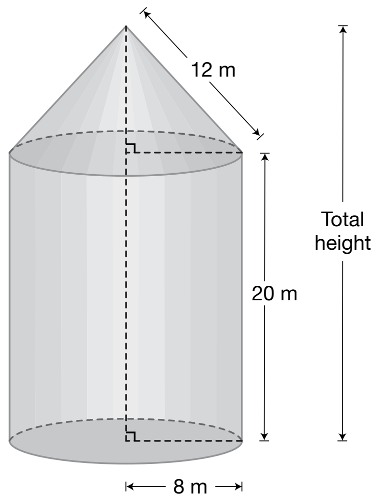
Understanding of Formulas

Students achieving Level 2 struggled to understand the relationship represented by a formula and to recognize when such a formula should be used. For example, when presented with a question that involved use of the side-length relationship for right triangles to find a missing value, only about 25% of students at Level 2 were able to correctly answer the question as compared to more than 80% of students at Level 3. Even though the formula ($a^2 + b^2 = c^2$) was provided within the question, students who answered incorrectly either did not use the formula accurately (i.e., double and divide by two instead of squaring and determining the square root) or used additive thinking to solve for the unknown height of the cone. Example 4 illustrates this issue and explores the additive thinking, instead of algebraic reasoning, students used in their option selection.

Example 4

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 30)

The dimensions of a silo are given.



Hint: Use the side-length relationship for right triangles, $a^2 + b^2 = c^2$.

Which value is closest to the **total** height of the silo?

A 24 m

B 29 m

C 32 m

D 34 m

Objective: Students were asked to find the total height of a silo and were given the formula for the side-length relationship for right triangles as a hint.

Overall Curriculum Expectation: E1, Geometric and Measurement Relationships. To answer correctly, students need an "understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations" (Ontario Ministry of Education, 2022, p. 176).

Mathematical Misunderstanding: The correct response for this question, option B, involves use of the formula for the side-length relationship for right triangles to solve for the total height. However, many students at Level 2 selected option C, which indicates that they added 20 m to 12 m. This reveals a reliance on additive thinking instead of recognizing that the formula provided could be used to find the missing value and then solving for the total height of the silo. These results point to the importance of students developing a conceptual understanding of the relationship represented by geometric formulas, and the use of the relationship between the lengths of the sides in a right triangle.

Comparison of Various Representations of Relations

Comparing various representations such as graphs, tables of values and equations for linear and non-linear relationships was also an area of improvement for students at Level 2. For example, when presented with questions that asked students to select which graph represents a given equation of a line, students achieving Level 2 often selected options that indicated they had difficulty making accurate connections between the different representations. This area for improvement also appeared in comparisons of equations. When presented with questions that asked students to determine, from a given set of equations, which equation is **not** an equation of a line, struggling students tended to select options that indicated they were unfamiliar with equations that were not written in $y = ax + b$ form.

Implications and Recommendations for Algebraic Reasoning

By exploring the properties of numbers and then developing an understanding of patterns, students build the generalization rules of algebra (Ontario Ministry of Education, 2013). In addition to generalization, algebraic reasoning also includes an understanding of unknowns expressed as variables, as well as a conceptual understanding of formulas and equivalence (Ontario Ministry of Education, 2013). The examples explored above illustrate the difficulties students had in these areas and highlight the importance of developing algebraic reasoning as students progress in the learning of mathematical concepts. Without consistent skills in algebraic reasoning, many algebra problems (including simplification) become more difficult. These skills are needed for simplifying an expression or an equation, making comparisons between equations and solving an equation for an unknown or missing value.

Recommendations

- Develop students' conceptual understanding in algebra by engaging them with mathematical ideas and the relationships between these ideas. This can include tasks in making conjectures, justifying or proving, and predicting (Ontario Ministry of Education, 2013).
- Use physical manipulatives to allow students to develop a concrete representation of algebraic ideas (Ontario Ministry of Education, 2013). For example, educators can consider using algebra tiles to represent variables and constants, thus allowing students to model and solve equations visually. With manipulatives, students can first apply operations concretely through the simplification of expressions and the collection of like terms. Then, they can transfer their concrete skills to pictorial representations and, finally, move to more abstract representations by using algebraic symbols (Fyfe et al., 2014). This three-step progression allows students to build a repository of memorable images to reference when abstract concepts become more difficult to understand (Fyfe et al., 2014).
- Consider providing opportunities for students to discuss and analyze algebraic problems that have already been solved so they can compare different solution strategies and the mathematical reasoning involved (Star et al., 2015).

Theme 3. Computational Thinking

Computational thinking is explored in our next theme, which can be defined as "the thought process involved in expressing problems in such a way that their solutions can be reached using computational steps and algorithms" (Ontario Ministry of Education, 2020b, p. 513).

When examining the option choices of students at Level 2, it became clear that students struggled with multi-step questions in general, and particularly those that involved operations with powers and exponent rules. The examples below explore these findings in more detail.

Operations with Powers

Students at Level 2 experienced some difficulty with questions that involved operations with powers, especially those with rational numbers. Approximately 20% of students achieving Level 2 answered this type of question correctly as compared to approximately 65% of students achieving Level 3. Example 5 illustrates the options selected by students who struggled in this area.

Example 5

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 6)

Select the expression that has the value that is the least.

A $\frac{1}{4}\left(\frac{(4^3)^2}{4^4}\right)$

B $\frac{1}{2}\left(\frac{4^3}{4^4}\right)^2$

C $\frac{1}{2}\left(\frac{(4^3)^2}{4^4}\right)$

D $\frac{1}{4}\left(\frac{4^3}{4^4}\right)^2$

Objective: Students were asked to select the expression that has the value that is the least.

Overall Curriculum Expectation: B2, Powers. To answer correctly, students need to be able to "represent numbers in various ways, evaluate powers, and simplify expressions by using the relationships between powers and their exponents" (Ontario Ministry of Education, 2022, p. 87).

Mathematical Misunderstanding: The correct option for this question (D) requires students to evaluate each expression and to understand operations with both exponents and fractions. Students who struggled with this question were more likely to select option A, indicating that they may have determined that one fourth is less than one half and therefore eliminated options B and C, but misunderstood how the brackets and exponents interact with the fraction. This example illustrates not only student misconceptions with powers but also how each of the mathematical themes discussed in this study interconnect with each other within the Grade 9 mathematics curriculum, as the question requires an understanding not only of power rules but also of order of operations and operations with mixed numbers (proportional reasoning).

Simplifying or Solving Expressions Involving Powers with Negative Bases

Students achieving Level 2 also struggled with simplifying expressions involving powers with negative bases, especially within the context of determining the sign (positive or negative) of the final answer. When presented with questions that required the evaluation of a power with a negative base such as –3 to the exponent 3, students at Level 2 who struggled in this area incorrectly evaluated the value of the power as 27 instead of –27, indicating that evaluating powers with negative bases is an area requiring conceptual strengthening.

Multi-Step Calculations

Analysis of option selections revealed that students at Level 2 struggled with questions that required not only an understanding of the mathematical concepts but also with planning sequential steps for questions that required multiple calculations to answer correctly. These types of questions are generally mapped to the Thinking category of Knowledge and Skills as adapted from the achievement chart for mathematics found in *The Ontario Curriculum, Mathematics Grade 9, De-Streamed* (*MTH1W*) (Ontario Ministry of Education, 2022). Example 6 illustrates these concepts below.

Example 6

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Thinking Example)

This table of values shows information about the linear relationship between the cost of an activity and the number of people participating.

Number of people	Cost of activity ($)
10	250
20	450
40	850

If the total cost for the activity is $750, how many people participate?

A 25 people

B 30 people

C 35 people

D 37 people

Objective: Students were presented with a table of values that shows the number of people participating in an activity and the associated cost of the activity. Students were then asked to determine the number of people participating given a cost not directly referenced in the table.

Overall Curriculum Expectation: C4, Characteristics of Relations. To answer correctly, students need an "understanding of the characteristics of various representations of linear and non-linear relations, using tools, including coding when appropriate" (Ontario Ministry of Education, 2022, p. 141).

Mathematical Misunderstanding: When presented with questions of this nature, students who answered incorrectly often selected options that indicated they had completed only one or some of the steps required to correctly solve the question or selected an option that revealed they had conducted the needed calculations in an incorrect sequence. In this example, the correct option is C, in which students calculated the slope or rate of change by calculating (450 – 250)/(20 – 10) = 20 or by determining it costs $200 for every 10 people, then determining the initial value (y-intercept) by calculating 250 – 200 = 50. Finally, the student might have used the equation $y = ax + b$, and substituted the determined or known values to solve for x ($750 = 20x + 50$), to arrive at 35. There is more than one way to solve this question; however, common misunderstandings involved students selecting option D, in which they may have estimated that the solution is slightly less than 40 people, or divided 750 by 20 as the number of people is increasing by 20 (750/20 = 37.5).

Implications and Recommendations for Computational Thinking

Students develop computational thinking skills by analyzing problems, identifying patterns, designing algorithms and testing solutions (Aho, 2012; Grover & Pea, 2013; Weintrop et al., 2016; Wing, 2006). These skills have many instructional implications since they are essential for success in science, technology, engineering and mathematics (STEM) and related fields (Aho, 2012; Grover & Pea, 2013; Weintrop et al., 2016; Wing, 2006). One of these computational thinking skills is problem solving, which allows students to break down more complex mathematical problems into smaller and more manageable parts (decomposition) (Wu et al., 2024). The examples explored above indicate that these decomposition skills are a growth area for students achieving Level 2.

Recommendations

- Make the process of problem solving explicit by discussing the thinking process with students (Ontario Ministry of Education, 2020a). Encourage students to share their thinking process by exploring the structure of a multi-step problem to determine what information has been provided and what information is needed, and to compare different solution strategies (Ontario Ministry of Education, 2020a).
- Regularly review and offer practice of difficult concepts such as order of operations and inverse operations. This practice should not only revisit students' prior knowledge but allow students to make connections between the skills, concepts and processes (Ontario Ministry of Education, 2020a).
- Support students to reflect on mistakes and the revision of their solutions (Ontario Ministry of Education, 2022). For example, have students reflect on how inverse operations and the substitution of a solution into an equation can be used as strategies to show reasoning.

Theme 4. Mathematical Language

The final theme explored in this report is mathematical language, which can be defined as "the conventions, vocabulary, and terminology of mathematics" (Ontario Ministry of Education, 2020b, p. 548). When examining the option choices of students at Level 2, strengths were identified in mathematical language. This included an understanding of the concepts of saving, appreciation and depreciation within financial situations. For example, when presented with a graphical representation of a declining monetary value over time, students correctly identified the situation as depreciation. Students also understood the concept of a balanced budget. For example, students could select a balanced budget when different budgets with expenses and income were presented in various ways to illustrate changes in financial circumstances.

It also became apparent through the analysis that students at Level 2 struggled with questions that required knowledge and understanding of the mathematical content and associated mathematical language within some questions. This occurred across strands and was found for certain concepts and terminology within the five-number summary for box plots, symbols and terminology used in graphing relationships, financial literacy, rational and irrational number sets and coding terminology. Each of these is explored below in more detail.

Five-Number Summary for Box Plots

Students achieving Level 2 struggled with questions involving the five-number summary (minimum value, first quartile, median, third quartile and maximum value) for box plots as outlined in the Data strand of the Grade 9 mathematics curriculum. When presented with questions that asked students to select a box plot that correctly displayed the median or quartiles based on a list of numbers, students who struggled in this area selected options that indicated some misunderstanding between median and mean and identifying the quartiles.

Symbols and Terminology Used in Graphing Relationships

Students achieving Level 2 also struggled with questions that required a conceptual understanding of some of the terminology, symbols and notation used to graph relationships. Examples include the use of inequalities, specifically $<$, $>$, $\leq$ and $\geq$.

Financial Literacy

The use of certain financial literacy terms, such as those used to describe interest rates, was also a growth area for students at Level 2. For example, comparing the effects of different types of interest (simple and compound) and compounding periods (e.g., annually, biannually) was an area for potential growth.

Rational and Irrational Number Sets

Students also struggled with the concept of number sets and how some are nested within each other. As Example 7 below illustrates, students at Level 2 struggled with identifying rational and irrational numbers. Approximately 50% of students at Level 2 answered this type of question correctly as compared to about 75% of students at Level 3.

Example 7

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 2)

Which statement about rational numbers is correct?

A. A rational number is always positive.

B. A rational number can be written as a fraction.

C. The square root of any number is a rational number.

D. A rational number in its decimal form is a non-ending, non-repeating decimal.

Objective: Students were asked which statement about rational numbers is correct and were presented with options that require a conceptual understanding of rational numbers.

Overall Curriculum Expectation: B1, Development of Numbers and Number Sets. To answer correctly, students need an "understanding of the development and use of numbers, and make connections between sets of numbers" (Ontario Ministry of Education, 2022, p. 78).

Mathematical Misunderstanding: When presented with questions of this nature, students who answered incorrectly often selected options that indicate they do not have a thorough understanding of the differences between rational and irrational numbers. In the analysis of option selections, students at Level 2 often selected option C, as opposed to the correct response, B. Students who struggled with questions of this type may not have a good understanding of how the various subsets of a number system are defined. This understanding is important when students explore why certain number sets have similarities while others do not.

Coding Terminology

The last example to be discussed illustrates students' misunderstanding of terminology needed for coding within the Algebra strand. An analysis of option selections revealed that for students achieving Level 2, only about 60% answered this type of question correctly as compared to 85% of students achieving Level 3. Example 8 illustrates the type of question encountered by students and the erroneous options often selected.

Example 8

(Resource: Released Questions, Grade 9 Assessment of Mathematics, Question 13)

When creating pseudocode, Suchen uses this expression.

distance > (8 * **minutes** + 15)

What is **distance** an example of in this pseudocode?

A a variable

B an equation

C a constant

D an inequality

Objective: Students were asked what 'distance' is within the context of the pseudocode.

Overall Curriculum Expectation: C2, Coding. To answer correctly, students need to "apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands" (Ontario Ministry of Education, 2022, p. 121).

Mathematical Misunderstanding: When presented with questions like this example, students who answered incorrectly often selected options that indicate they do not have a full conceptual understanding of the algebraic concept of a variable. The correct option is A (distance is a variable); however, our analysis found that students often chose C (distance is a constant). EQAO uses pseudocode in questions as per the coding examples in the teacher support material as outlined in the Grade 9 mathematics curriculum. These questions include those mapped to expectation C2 and use coding to demonstrate an understanding of algebraic concepts including variables, parameters, equations and inequalities. As such, a misunderstanding of the concepts of 'constant' and 'variable' are important aspects not only within Coding but also within the rest of the Algebra strand.

Implications and Recommendations for Mathematical Language

As mentioned above, students at Level 2 struggled with rational and irrational number sets, the five-number summary for box plots, financial literacy terms, coding terminology and the symbols and terms used in graphing relationships. Mathematical language and vocabulary are an important component of student understanding of mathematical concepts and the connections between them (Riccomini et al., 2008). Mathematical vocabulary has been linked to mathematical performance (Lin et al., 2021) and is emphasized in the Grade 9 mathematics curriculum, as "teachers need to explicitly teach mathematical vocabulary, focusing on the many meanings and applications of the terms students may encounter" (Ontario Ministry of Education, 2022, p. 50).

Recommendations

- Encourage students to use mathematical language in class and provide feedback on its use in written and oral forms. This can provide opportunities for students to build connections to mathematical terms through classroom discourse.
- Use word walls, which can be effective in supporting students as new mathematical concepts and words are introduced (Hattie et al., 2017). For example, consider using a variety of representations for number sets and their subsets, which allows for student discussion, questions and exploration.
- Introduce new vocabulary via direct instruction in ways that activate students' prior knowledge (Ontario Ministry of Education, 2020a). For example, students' prior knowledge in budgeting within the Financial Literacy strand could be used to connect to new learning regarding different interest rates.

Conclusion

Through the analysis of question responses on the 2022–2023 Grade 9 Assessment of Mathematics, patterns of students' mathematical understanding and misconceptions were discovered and categorized into four themes: Proportional Reasoning, Algebraic Reasoning, Computational Thinking and Mathematical Language. By grouping mathematical understanding into these four areas, student strengths and areas for growth were identified across the different mathematical curriculum strands. The themes identified in this study revealed opportunities for growth in mathematical understanding and helped us categorize recommendations that can be applied systematically, as the skills and concepts discussed are closely connected. For example, improving student ability to solve multi-step problems should not be approached without also addressing difficulties in proportional and algebraic reasoning. Likewise, building mathematical vocabulary is an important activity that can be reinforced across strands.

Other important factors, such as knowing the learner and the curriculum, providing a positive and safe learning environment, using various assessment methods and fostering perseverance in mathematical learning, are also essential aspects of effective mathematical instruction (Ontario Ministry of Education, 2020a). These elements should be included in a systematic approach to improving mathematical proficiency. This report supports educators in getting to know their learners by providing a detailed analysis of the mathematical strengths and growth opportunities for students at Level 2. Educators can further tailor this analysis by using EQAO's released questions within their classrooms to assess current student understanding. Examining student answers to the released questions can uncover possible misconceptions at the student level and inform instructional approaches that address the specific needs of students.

Notes

[1] The assessment does not measure curriculum content from the Mathematical Thinking and Making Connections strand or the Social Emotional Learning (SEL) Skills in Mathematics strand.

References

Aho, A. V. (2012). Computation and computational thinking. *The Computer Journal, 55*(7), 832 – 835. https://doi.org/10.1093/comjnl/bxs074

Education Quality and Accountability Office. (2020). *Leveraging multi-stage computer adaptive testing for large-scale assessments.* https://www.eqao.com/wp-content/uploads/2021/01/msCAT-literature-review.pdf

Education Quality and Accountability Office. (2023a). *Grade 9 assessment of mathematics framework.* https://www.eqao.com/wp-content/uploads/2020/10/Framework-grade9-assessment-of-mathematics.pdf

Education Quality and Accountability Office. (2023b). *Resource: Released questions, grade 9 assessment of mathematics.* https://www.eqao.com/wp-content/uploads/2023/11/math-resource-released-questions-g9-2023.pdf

Education Quality and Accountability Office. (2023c). *Student achievement on mathematics curriculum strands: Results from the 2021–2022 primary- and junior-division and grade 9 assessments.* https://www.eqao.com/wp-content/uploads/2024/02/eqao-2023-mathematics-strands-report.pdf

Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2014). Concreteness fading in mathematics and science instruction: A systematic review. *Educational Psychology Review, 26,* 9–25. https://doi.org/10.1007/s10648-014-9249-3

Grover, S., & Pea, R. (2013). Computational thinking in K–12: A review of the state of the field. *Educational Researcher, 42*(1), 38–43. https://doi.org/10.3102/0013189X12463051

Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics, grades K–12: What works best to optimize student learning.* Corwin Press.

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers (3rd ed.).* Routledge.

Lin, X., Peng, P., & Zeng, J. (2021). Understanding the relation between mathematics vocabulary and mathematics performance: A meta-analysis. *Elementary School Journal, 121*(3), 504–540. https://doi.org/10.1086/712504

Ontario Ministry of Education. (2012). *Paying attention to proportional reasoning: Support document for paying attention to mathematical education.*

Ontario Ministry of Education. (2013). *Paying attention to algebraic reasoning: Support document for paying attention to mathematics education.*

Ontario Ministry of Education. (2020a). *High-impact instructional practices in mathematics.* https://assets-us-01.kc-usercontent.com/fbd574c4-da36-0066-a0c5-849ffb2de96e/d7fbcc68-2f2b-4ef8-988d-3fb9b42c33cd/high-impact-instruction-math.pdf

Ontario Ministry of Education. (2020b). *The Ontario curriculum, grades 1-8: Mathematics, 2020.* https://assets-us-01.kc-usercontent.com/fbd574c4-da36-0066-a0c5-849ffb2de96e/90439c6e-f40c-4b58-840c-557ed88a9345/The%20Ontario%20Curriculum%20Grades%201%E2%80%938%20-%20Mathematics%2C%202020%20%28January%202021%29.pdf

Ontario Ministry of Education. (2022). *The Ontario curriculum, mathematics grade 9, de-streamed (MTH1W), 2021.* https://assets-us-01.kc-usercontent.com/fbd574c4-da36-0066-a0c5-849ffb2de96e/c292051c-00a6-4f34-9778-39c1427281e0/The%20Ontario%20Curriculum%20-%20Mathematics%20Grade%209%20De-streamed%20Course%202021_with%20Teacher%20Supports.pdf

Riccomini, P. J., Sanders, S., & Jones, J. (2008). The key to enhancing students' mathematical vocabulary knowledge. *Journal on School Educational Technology, 4*(1), 1–7. https://www.learntechlib.org/p/175569/

Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R., Lyskawa, J., McCallum, W. G., Porath, J., & Zbiek, R. M. (2015). *Teaching strategies for improving algebra knowledge in middle and high school students* (NCEE 2014-4333). Institute of Education Sciences. https://ies.ed.gov/ncee/WWC/Docs/PracticeGuide/WWC_Algebra_PG_Revised_02022018.pdf

Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication … and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*(3), 360–381. https://doi.org/10.1080/07370008.2010.488306

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology, 25,* 127–147. https://doi.org/10.1007/s10956-015-9581-5

Wing, J. M. (2006). Computational thinking. *Communications of the ACM, 49*(3), 33–35. https://doi.org/10.1145/1118178.1118215

Wu, T. T., Asmara, A., Huang, Y. M., & Hapsari, I. P. (2024). Identification of problem-solving techniques in computational thinking studies: Systematic literature review. *SAGE Open, 14*(2). https://doi.org/10.1177/21582440241249897