EQAO

Student Achievement on Mathematics Curriculum Strands, 2021–2022

Implications for Instructional Planning

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Summary of implications

The findings of this study, which explored student achievement on mathematics strands, indicated that it is essential to address mathematics ability across all curriculum strands, as mathematical thinking and skills are highly connected. Indeed, the importance of the integration of strands is reflected in the provincial report cards for Grades 1 to 8, where students receive one overall mark for mathematics. Educators may therefore choose to integrate the curriculum strands through a planning model that permits them to intentionally integrate different concepts and strands (e.g., through the spiral approach or interleaving) in the classroom throughout the school year. In this way, educators can gradually and systematically introduce more complex and rigorous tasks, and the progression of learning will become more complex both as the school year progresses, and from grade to grade.

For example, educators might introduce a topic that students typically struggle with at the start of the school year and teach a mini-unit to teach the math facts associated with the topic. Then in a month, they might layer the use of these math facts into slightly deeper applications or activities that can be accessed by students at all levels. The topic could then be seen again a month or two later, integrated into or with another strand to support students' understanding that all mathematics is connected. Through this "spiralled" approach, there is also opportunity to use high-impact strategies such as flexible groupings to engage in mathematical conversations and problem-solving tasks and experiences.

That said, students' achievement in key curriculum strands at specific grade levels may flag students at risk of falling behind curriculum expectations. For example, according to the analyses in this study, it is likely that from Grades 1 to 3, students' progress in the Number strand is critical to ensuring they are able to maintain achievement in other curriculum strands as they move to the next grade level. Focusing on related Number concepts in kindergarten is likely also important. Indeed, the curriculum makes clear that aspects of the Number strand underlie other mathematical activities:

> Most students learn math facts gradually over a number of years, connecting to prior knowledge, using tools and calculators. Mastery comes with practice, and practice helps to build fluency and depth. Students draw on their ability to apply math facts as they manipulate algebraic expressions, equations, and inequalities.
>
> (*The Ontario Curriculum, Grades 1–8: Mathematics,* 2020)

As a concrete example, taking Grade 3 Number strand expectations as a learning target, we can see the development of multiplicative thinking begins in Grade 1 with the connection to problem solving through equal groupings (B2.5), which then develops into repeated equal groupings in Grade 2 (B2.5), and then into representing multiplication and division of values up to 10×10 (B2.6) and solving problems of grouped fractions (B2.7). Other curriculum expectations that contribute to this are the concept of skip counting in Grade 1 (B1.5), Grade 2 (B1.4) and Grade 3 (B1.4). The values being skip counted in Grade 1 and Grade 2 allow educators to make connections to repeated groupings and thus multiplication. Meanwhile, the development of multiplicative thinking draws on several co-occurring learned strategies (e.g., subitizing, counting all, counting groups, modelling composite units, skip counting) and key ideas (e.g., cardinality, unitizing, part-whole relations, proportional reasoning). These strategies and ideas all require that students constantly come back to a concept and revisit the idea with explicit instruction (Lawson, 2015). Like most teaching techniques, these strategies are not effective if they are aiding the completion of an activity only once or twice in a year.

Similarly, from Grade 4 to 6, grade-level progress in both Number and Algebra appears to be central to ensuring that students are able to meet curriculum expectations in other curriculum strands, and ensuring they are prepared for the mathematics curriculum in Grade 7 and beyond.

Another finding of note for the junior grades is that Grade 6 students in French-language boards are not performing as well on questions in the Financial Literacy strand as students in English-language boards. Unlike other strands, the Financial Literacy curriculum content at Grade 6 is highly definition based. This factor may be the source of the differences, but it is unclear why that is. Further work is needed to understand the reasons for these differences.

From Grade 7 to 9, ensuring students grasp key concepts for Algebra primarily, but also for Spatial Sense/Geometry and Measurement, appears to be critical to ensuring students are able to meet curriculum expectations in Grade 9. For example, in the Algebra expectations in Grades 7 to 9, several algebra concepts are further developed, introducing more complex inequalities and more complex linear equations to be solved. Moreover, from an instructional perspective, in Grade 9 Algebra, there is a shift in the methodology of how to teach and assess linear equations. The emphasis moves away from explicit and direct instruction of each character trait of a single form of a line (e.g., $y = mx + b$) and moves toward inquiry-driven learning into why equations, lines and graphs behave how they behave and what changes to the equation or pattern might cause each reaction or transformation. This shift allows for more conversation about how linear (e.g., $x + y = k$) and non-linear (e.g., $xy = k$) relationships are similar in their behaviours of movement as students engage in the secondary mathematics curriculum in Grades 10 to 12.[1]

In Geometry and Measurement, the Grade 9 curriculum moves away from evaluation processes, for example, calculations of perimeter, area, volume and surface area. In exchange, the curriculum becomes more focused on algebraic relationships (for example, how the volume of an object will be impacted if its height is doubled and its radius is tripled) and solving processes. Because of this, algebra skills are also essential for mastering Grade 9 Geometry and Measurement curriculum expectations.

[1] It is important to highlight that the 2020 curriculum introduced coding expectations in Algebra from Grades 1 to 9. The expectations become more text-based and focused on coding to solve mathematics problems by Grade 4; then in Grade 7 the concept of subprograms is introduced, followed by more advanced subprograms in Grade 9, which drastically increases the length of the code and therefore the amount of text to read.

Practical examples of how Number concepts undergird other strands in Grades 1 to 3

Given the findings of the study, it is proposed that the key implication for instructional planning in Grades 1 to 3 is to recognize that the Number strand is a key strand in the primary curriculum and covers many foundational mathematical concepts. For example, Grade 3 curriculum expectations for operations require students to represent the multiplication and division of numbers up to 10 by 10, and recall multiplication facts of 2, 5 and 10 and related division facts.[2] These abilities are required for expectations in several other strands. Therefore, integrating Number instruction with instruction on other strands is important throughout Grades 1 to 3. Several examples below are presented to assist educators in reflecting on the forms that integrated instruction might take.

Number and Algebra

Due to the foundational nature of concepts covered in Number, the strand impacts many concepts in the Algebra strand, as students must understand the concepts in the Number strand to understand the numeric representation found in the Algebra strand. For example, number skills such as skip counting are used in determining patterns (including in tables of values), and operation skills are utilized in creating pattern rules, as well as in creating and describing relationships in patterns. Therefore, if students have not yet mastered grade-level expectations for number and operation skills, they may also have difficulty understanding some concepts in the overall and specific expectations in the Algebra strand. Note also that variables are introduced through the Algebra strand, which can be an abstract concept for students to grasp.

In the 2020 curriculum, coding was also introduced to the Algebra strand. In the primary grades, coding curriculum expectations focus on the use of coding to solve mathematics problems or show mathematical patterns, so students' success in this topic may also be highly influenced by their understanding of the concepts in the Number strand. In addition, the third overall expectation in the Algebra strand includes utilizing code involving sequential movements, repeating operations (similar to creating a table of values), as well as altering code from one set of commands to another.

Number and Data

Once again, in the Data strand there are several concepts that rely on students having mastered concepts presented in the Number strand. For example, in the Data strand students are expected to determine the mean and identify the mode for a data set. They are also expected to construct graphs. The curriculum expectations in Data that focus on probability also require students to have mastered Number strand concepts, because determining a comparative likelihood requires fractional comparisons (e.g., few or many).

[2] It is important to highlight that the 2020 curriculum introduced some important changes to the Number strand. For example, this strand now introduces fractions earlier in more formal contexts, which changes when and how fractions are taught. Specifically, in Grade 3 the previous curriculum required students to use concrete materials without explicitly using fractional notation (and instead using, for example, halves, fourths and thirds). However, the new curriculum includes a wider range of fractions for students to master and includes the use of fractional notation. Students are introduced to fractions and encouraged to write them using fractional notation in Grade 3. By Grade 5, they are adding and subtracting fractions.

Number and Spatial Sense

Another example of the importance of Number concepts for other curriculum areas is found in the Spatial Sense curriculum expectations. The concepts in the Grade 3 overall Spatial Sense expectation that focus on measurement highly utilize operation skills outlined in the Number strand through the estimation and calculation of lengths, perimeters, masses, area, time and unit conversions.

In contrast, the Grade 3 Spatial Sense expectation that focuses on the visualization of two-dimensional shapes and three-dimensional objects is not influenced by number skills in the same way. However, it is important to note that strong visualization skills are related with high levels of achievement in mathematics in later years (Frick, 2019; Hawes et al., 2019; Moss et al., 2016; Wang, Hui & Zhang, 2021).

Number and Financial Literacy

Finally, the Financial Literacy strand in the Grade 3 curriculum has one overall and one specific expectation. This expectation states that students should learn to estimate and calculate change, which also requires number and operation skill sets. EQAO assessment questions mapped to this expectation are included in the reporting for the Number strand.

Practical examples of how Number and Algebra undergird other strands in Grades 4 to 6

In the junior mathematics curriculum, the Number and Algebra strands are highly connected to each other and together appear to take on a central role in students' successfully meeting grade-level expectations in Grade 6. In contrast, the Financial Literacy curriculum expectations require a vocabulary-based knowledge set that is broadly independent of the Number and Algebra strands. A significant introduction in the junior grades is that by Grade 6, students are required to start moving away from using concrete materials toward adopting diagrammatic and abstract representations.

The key implication of the analysis findings and the curriculum developments described above is that instructional planning in Grades 4 to 6 should ensure that the integration of strands positions both Number and Algebra curriculum expectations so that students are set up for success when learning concepts from Data and Spatial Sense strands. For Financial Literacy, it is recommended that educators ensure students can use language related to daily finances. Several examples are presented below to assist educators in reflecting on the forms that integrated instruction might take.

Number and Algebra

An example of the Grade 6 curriculum connections between the Number and Algebra strands is the link between

- the second overall expectation in the Number strand (B2. Operations) and
- the patterns and relationships section in the first overall expectation of the Algebra strand and
- the equations and inequalities section in the second overall expectation of the Algebra strand.

If students struggle to understand the computations of operations and the order of operations with whole numbers, integers and fractions, then accessing relationships where they are building tables and equations and solving equations through inverse operations will likely be challenging.

In addition, the coding component of the Algebra strand in Grade 6 often assumes a strong grasp of number sense and operations. This can be seen in Figure 1, with the use of a measurement conversion incorporating the multiplication of whole and decimal values into the problem.

Figure 1. The pseudocode from a sample task provided in the teacher supports for C3.1 of the 2020 mathematics curriculum for Grade 6

```
set metres = 0.00
set centimetres = 0.00
set convertMetres = 0.00
set convertCentimetres = 0.00
keyPressed = "blank"
repeat until keyPressed = "A" or "B"
    output "Type A to convert from metres (m) to centimetres (cm). Type B to convert from cm to m."
    store user input as keyPressed
if keyPressed == "A"
    output "What is your measurement in metres?"
    store user input as metres
    convertCentimetres = metres * 100
    output metres, "metres is", convertCentimetres, "centimetres."
else
    output "What is your measurement in centimetres?"
    store user input as centimetres
    convertMetres = centimetres * 0.01
    output centimetres, "centimetres is", convertMetres, "metres."
```

Number, Algebra, Proportional Reasoning and Data

There are two overall expectations in the Grade 6 Data strand: Data Literacy and Probability. The first overall expectation, Data Literacy, focuses on terminology, plus the collection and organization of data. These collection and organization activities include requiring students to construct different types of graphs. For this reason, success on the overall expectation for Data Literacy is not directly dependent on possessing strong number sense but does require students to understand scales.

The ability to understand scales is related to proportional reasoning and can influence students' ability to read and interpret graphs. It is also worth highlighting that proportional reasoning, including fraction knowledge and understanding of probability, is an important predictor of student success in mathematics (Hilton et al., 2016; Lesh, Post & Behr, 1988). Developing strong proportional reasoning skills can lead to better overall mathematics achievement, improved understanding of mathematical concepts and success in advanced mathematics (e.g., Booth & Newton, 2012; Taylor & Jones, 2009).

The second overall expectation in Data is Probability. Grade-level mastery of this topic relies on students' understanding of fractions, decimals, percentages, ratios and rates. Each of these five concepts is found in the Number strand in the curriculum and is often used as tools of proportionality along with proportional reasoning skills in the Algebra strand.

Number, Algebra and Spatial Sense

There are two overall expectations in the Grade 6 Spatial Sense strand: Geometric and Spatial Reasoning, and Measurement. Geometric and Spatial Reasoning includes mastering abstract visualizations, including geometric constructions from side views, as well as transformations of shapes on a four-quadrant Cartesian plane. While these two subtopics appear unrelated to number sense and algebra at first glance, the Cartesian plane is where relations can be graphed in the Algebra strand. This demonstrates that the concepts in the Number and Algebra strands impact understanding of concepts in the Spatial Sense strand.

The second overall expectation, Measurement, covers several subtopics, including the metric system, angles, and area and surface area. Each of the specific expectations requires students to determine how and when to apply their number sense and operation skills as well as many of their developing algebra skills. For example, determining the area of composite polygons requires students to decompose the shape into other known shapes and apply the various area formulae, use the order of operations, calculate the individual areas and then determine the sum or difference as the total area.

Financial Literacy

At Grade 6 there is one overall expectation for Financial Literacy and within it there are three subtopics: Money Concepts, Financial Management, and Consumer and Civic Awareness. The finance concepts covered in this curriculum are more definition based than calculation based. For this reason, students need to have an understanding of the language used in these topics. This means that the understanding exhibited within this strand is likely independent of students' knowledge of Number and Algebra strands.

Practical examples of how Algebra undergirds other strands in Grades 7 to 9

It is proposed that the key implication of the results of the strands study is that instructional planning in Grades 7 to 9 should recognize that the Algebra strand is a key strand in the intermediate and Grade 9 curricula as students develop their mathematics skills toward increasingly advanced, abstract mathematical concepts. The examples below show that students' developing Algebra skills support continued progress in learning on other strands.

Number

The Number strand in Grade 9 relies less on numbers and operations than it does at the primary and junior stages. Instead, Number strand learning in Grade 9 returns to an introductory level and understanding of new concepts is expanded upon in future mathematics courses. As a result, some Number expectations shift in Grade 9 to various concepts that may not directly impact the other strands. For example, in Number the specific expectation B1.2 requires students to define subsets of number systems and describe differences, while specific expectation B1.3 requires students to discuss number relationships and patterns to understand density of number, infinity and limit.

However, some of the concepts in the overall expectations in Number do carry forward into topics covered in other strands, where they are learned in specific contexts and through particular uses. For example, the content in specific expectation B3.3 requires students to explain the effects of positive and negative signs on ratios, rates, fractions and decimals, and mastering this expectation impacts students' ability to master concepts in the Algebra strand. For example, expectation C1.2 in the Algebra strand specifies that students must be able to create algebraic expressions to generalize relationships expressed in words, numbers and visual representations, in various contexts. Additionally, expectation C1.5 specifies that students must be able to create and solve equations in various contexts and verify their solutions.

Finally, it is worth noting that in the Grade 9 course, some students will demonstrate difficulty with fractions, with how fractions behave in mathematical operations and with order of operations tasks in general. Such students may also have difficulty using algebraic equations and expressions both with and without a context. For example, the specific expectation in the algebra strand C1.5 asks students to create and solve equations in various contexts.

Algebra

The Algebra strand has four overall expectations in Grade 9: Algebraic Expressions and Equations, Coding, Application of Relations, and Characteristics of Relations. A key feature of the Algebra concepts covered in Grade 9 is that students must use fewer concrete manipulatives and diagrammatic representations, and instead create standard understandings through abstraction by the end of the course. Meeting grade-level expectations for this shift is a keystone aspect of meeting Grade 9 mathematics curriculum expectations.

In Grade 9, coding becomes more advanced. This can be seen in Figure 2. The pseudocode listed in the main program asks for information to create the equation for each of two lines and runs a subprogram after getting the information for the line. The subprogram plots 10 sets of coordinate points for the line. To be able to read and interpret this sequence of code, students need to understand how to use the order of operations to determine each *x* and *y* value in the list of plotted points being determined. Even though to answer questions about the code, some students might not need to complete the calculations, many students employ a strategy of using an example of values to test the pseudocode to complete the instructions. Not having a mastery of number sense would mean that students cannot access this strategy to support their learning of coding.

Figure 2. The pseudocode from a sample task provided in the teacher supports for C2.1 of the 2021 Grade 9 mathematics curriculum for MTH1W

Main program

initialValue = 3
rateOfChange = 2
totalPoints = 10
run **plotRelation** subprogram (**initialValue**, **rateOfChange**, **totalPoints**)
initialValue = 0
rateOfChange = 3
totalPoints = 10
run **plotRelation** subprogram (**initialValue**, **rateOfChange**, **totalPoints**)

Figure 2 (continued). The pseudocode from a sample task provided in the teacher supports for C2.1 of the 2021 Grade 9 mathematics curriculum for MTH1W

plotRelation subprogram

```
subprogram plotRelation (initialValue, rateOfChange, totalPoints)
    xValue = 0
    yValue = initialValue
    repeat totalPoints times
        plot point (xValue, yValue)
        xValue = xValue + 1
        yValue = rateOfChange * xValue + initialValue
```

Pseudocode does not represent a specific programming language. It can be adapted to work with a variety of programming languages and/or environments.

Algebra and Data

In the Grade 9 Data strand, there are two overall expectations: Collection, Representation and Analysis of Data; and Mathematical Modelling. Collection, Representation and Analysis of Data has been spiralled through the primary and junior grades, and thus in Grade 9 this overall expectation is about strengthening students' data management and interpretation skills.

However, Mathematical Modelling emphasizes connections between various concepts in the Algebra strand and mathematical models. For example, students are required to be able to use the line of best fit of a scatter plot to determine the linear equation and analyze a situation. Therefore, previous mastery of grade-level Algebra expectations directly underlies students' ability to meet grade-level expectations for Mathematical Modelling.

Algebra and Geometry and Measurement

In the Grade 9 Geometry and Measurement strand, there is a shift away from spatial sense and a new focus on solving problems in geometric and measurement relationships. Solving these types of problems, descriptions of which are given below, requires a solid understanding of concepts in the Algebra strand.

Students solve problems involving

- relationships between the volumes of various objects, and show how changing one or more dimensions of a shape or object affects perimeter/circumference, area, surface area or volume.
- different units within a measurement system and between measurement systems.
- the side-length relationship for right triangles.
- determining unknown angles (e.g., creating and solving algebraic expressions to determine missing angles).

All these problems require an understanding of the skills and concepts in the Algebra strand.

Financial Literacy

In Grade 9, there is one overall expectation for Financial Literacy regarding financial decisions. Students are required to consider financial situations that involve appreciation, depreciation, comparing the effects of various aspects of borrowing and lending and modifying budgets. Some parts of the learning in this strand are independent of students' knowledge of Number and Algebra strands.

Conclusion

In summary, there are three key considerations that have emerged from cross-referencing the curriculum to the Grades 3, 6 and 9 EQAO numeracy assessment results from the 2021–2022 school year. It is important to recognize that the curriculum in each grade is interwoven across the strands, with more significant cross-strand connections within Number and Algebra. There is a shift in focus within the curriculum as a whole from Grade 1 to 9, as well as within single grades, from learning concepts with the concrete (e.g., manipulatives) through phases toward abstraction. There is significant importance placed on students' conceptual understanding and skills concerning Number in Grade 3 that, as students progress, shifts to share emphasis between Number and Algebra in Grade 6, and shifts again to Algebra and Geometry and Measurement in Grade 9.

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