Framework

Grade 9
Assessment
of Mathematics

Education Quality and
Accountability Office

EQAO

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Contents

Introduction

This framework provides a detailed description of the EQAO Grade 9 Assessment of Mathematics, which is conducted each year in Ontario. It also describes how the Grade 9 assessment aligns with the expectations in *The Ontario Curriculum*.

Who is this framework for?

This framework has been prepared for

- educators;
- parents and
- members of the general public.

What is in the framework?

In this framework, you will find

Chapter 1: a brief introduction to EQAO, large-scale assessments in Ontario and the Grade 9 Assessment of Mathematics, and information on the differences between large-scale and classroom assessment.

Chapter 2: information about the purpose and benefits of the Grade 9 Assessment of Mathematics and a description of how results are reported.

Chapter 3: an introduction to the mathematics assessment and how it links to *The Ontario Curriculum* and aligns with current research.

Chapter 4: discussions of the assessment process, the content of the assessment booklets, how EQAO ensures that English language learners and students with special education needs can participate fairly and the meaning of Ontario's achievement levels.

Chapter 5: the assessment blueprint and information on how EQAO assessments are aligned with curriculum expectations.

Chapter 6: information on how student responses to mathematics questions are scored.

Chapter 7: a discussion of how EQAO ensures that its assessments are comparable from year to year.

CHAPTER 1: About EQAO and Provincial Assessments

In This Chapter

- What is EQAO?
- What is assessment?
- What assessments does EQAO conduct?
- What is the Grade 9 Assessment of Mathematics?

Insight: Differences between large-scale and classroom assessment

WHERE TO LEARN MORE

For more information and valuable resources for parents and educators, visit the EQAO Web site:

www.eqao.com

What is EQAO?

The Education Quality and Accountability Office (EQAO) is an arm's-length agency of the provincial government that measures the achievement of students across Ontario in reading, writing and mathematics, and reports the results to parents, educators and government. EQAO assessments are based on the expectations in *The Ontario Curriculum*.

EQAO results are reported at the provincial, school board and school levels. They are used by the Ministry of Education, district school boards and schools to improve learning and teaching and improve student achievement. An Individual Student Report is also provided by EQAO for each student who writes an EQAO assessment.

What is assessment?

Assessment is an important part of teaching and learning. For example, teachers use assessment in the classroom to gauge the skills and knowledge of their students. They use this information to plan their teaching and identify individual students who may need additional help.

A traditional test is one kind of assessment, but student progress can be measured in many other ways. Reviewing a portfolio of student work is one example. Large-scale assessments, like those conducted by EQAO, measure student achievement across the province at critical times in students' school careers.

Insight:

Differences between large-scale and classroom assessment

EQAO's Large-Scale Assessments	Classroom Assessment
The purpose of EQAO's large-scale assessments is to provide comparable year-to-year data to give the public information on student achievement.	The purposes of classroom assessment are to improve student learning (using models such as Ministry exemplars to assess the quality of work), to report regularly on student achievement and to provide timely, constructive feedback for improvement.
EQAO's large-scale assessments provide reliable, objective and high-quality data that can inform school boards' improvement planning and target setting.	Classroom assessments encourage students to engage in self-evaluation and personal goal setting. They also provide parents with information on strengths and weaknesses that can be used to encourage improvement.
EQAO's large-scale assessment materials are created and scored "at a distance." The assessment scorers do not know the students personally.	Classroom assessment materials are usually created and marked by a teacher who knows the students personally.
EQAO's large-scale assessments are summative; they present a snapshot of student achievement or learning at the time the assessment is administered.	Classroom assessments are conducted in an instructional context and include diagnostic, formative and summative assessment. They are administered at regular intervals over time.
EQAO's large-scale assessments require students to demonstrate their knowledge and skills independently on standardized tasks and under standardized conditions, although some accommodations are allowed for students with special education needs.	A wide variety of supports (reminders, clarification) are often available to address students' special education needs and abilities.

EQAO's large-scale assessments measure achievement against expectations from the prescribed curriculum and contain tasks and items that sample from and represent the curriculum for the domain assessed.	Classroom assessments measure expectations from the curriculum and contain tasks and items that represent expectations, topics and themes that have been taught. The questions are written in language used regularly in the classroom by the teacher.
EQAO's large-scale assessments provide the same (in a given year) or psychometrically comparable items (from year to year) for all students.	Classroom assessments can provide modified items or tasks tailored to the special education needs of individuals or groups of students.
In order for students' results on EQAO's large-scale assessments to be comparable across the province, the assessments must be administered, scored and reported on in a consistent and standardized manner.	Results of classroom assessments across the province are not always comparable, because of the variation in administration procedures and time allowed, amount of teacher support, modification of items to suit student needs and teacher autonomy in marking.
For EQAO's large-scale assessments, all scorers use the same scoring guides and are trained and monitored to ensure objectivity and consistency.	The marking of classroom assessments is more subjective and is often influenced by contextual information about the students that is available to the teacher. Teachers use the achievement charts in the curriculum policy documents to guide assessment decisions.

What assessments does EQAO conduct?

EQAO conducts four provincial assessments each year. These are

- the Assessment of Reading, Writing and Mathematics, Primary Division (Grades 1–3);
- the Assessment of Reading, Writing and Mathematics, Junior Division (Grades 4–6);
- the Grade 9 Assessment of Mathematics and
- the Ontario Secondary School Literacy Test.

What is the Grade 9 Assessment of Mathematics?

The Grade 9 Assessment of Mathematics, which is the subject of this framework, evaluates the knowledge and skills that *The Ontario Curriculum* expects students to have learned by the end of Grade 9. The assessment is used to determine how well students are achieving these expectations, and their level of achievement. See Chapter 4 for more information about Ontario's student achievement levels.

In This Chapter

- What is the purpose of the Grade 9 Assessment of Mathematics?
- What is reported?
- What are the benefits of the assessment?

CHAPTER 2: The Grade 9 Assessment of Mathematics

What is the purpose of the Grade 9 Assessment of Mathematics?

The purpose of the Grade 9 Assessment of Mathematics is to assess the level at which students are meeting curriculum expectations in mathematics up to the end of Grade 9.

The results of this assessment are reported by
- individual student;
- school;
- school board and
- the whole province.

What is reported?

The Individual Student Report includes
- the student's overall results.

School reports include
- overall school-level results, with comparisons to board and provincial results;
- results by subgrouping, such as by gender and English language learner and special needs status;
- contextual data on demographics and participation in the assessment;
- results over time;
- results of the student questionnaire accompanying the assessment;
- a Student Roster report that shows individual student results on sample assessment items, with overall board and provincial results for comparison, and
- profiles of strengths and areas for improvement in mathematics.

Board reports include
- overall board results, with comparisons to provincial results;
- contextual data, results over time, reports by subgroup and questionnaire data and
- profiles at the board level of strengths and areas for improvement in mathematics.

Provincial reports include
- overall provincial results, including results by board;
- contextual data, results over time, reports by subgroup and questionnaire data;
- instructional strategies for success and
- case studies (school success stories).

Note: In cases where the number of students being reported on for a school or board is small enough that individual students could be identified, EQAO does not release the reports publicly.

What are the benefits of the assessment?

EQAO provides the Ontario school system with valid, reliable and comparable year-to-year data on student achievement. Schools and boards can confidently use this data along with other contextual and assessment information (e.g., on demographics, attendance and pass rates) to determine how well their improvement strategies, such as staff development or new learning resources, are working.

Beyond specific reporting, the assessment
- provides data to assist schools and boards in improvement planning and target setting;
- supports the successful implementation of the curriculum;

- improves understanding of assessment practices and the curriculum levels of achievement among educators across the province and
- improves understanding of assessment practices among the public.

CHAPTER 3: What Is Assessed

In This Chapter

- What is the definition of mathematics for the assessment?
- How does EQAO's Grade 9 assessment link to *The Ontario Curriculum*?
- What is assessed?

Insight: How the definition of mathematics aligns with current research

What is the definition of mathematics for the assessment?

A number of sources have been used to construct the following definition of mathematics.

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (Organisation for Economic Co-operation and Development, 2003).

Achievement in mathematics goes beyond knowing mathematical facts and procedures; it also means being able to reason mathematically and to have the ability to interpret and solve mathematical problems (Artelt, Baumert, Julius-McElvany & Peschar, 2003).

Content Strands

Mathematics spans several content strands or domains. Strands such as Number Sense, Geometry, Measurement, Algebra, Statistics and Probability can easily be matched across elementary school mathematics curricula from most countries and provinces.

In Ontario, the Grade 9 content strands Number Sense and Algebra; Linear Relations; Measurement and Geometry; and Analytic Geometry reflect those of the elementary program, as shown in the following table:

Elementary Strands		Grade 9 Strands
Number Sense and Numeration	lead to →	Number Sense and Algebra
Measurement		Linear Relations
Geometry and Spatial Sense		Analytic Geometry (academic only)
Patterning and Algebra		Measurement and Geometry
Data Management and Probability		

Mathematical Processes

Mathematics involves many different processes. It is often defined as having the following five components:

- *Conceptual understanding*—comprehension of mathematical concepts, operations and relations
- *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently and appropriately
- *Strategic competence*—ability to formulate, represent and solve mathematical problems
- *Adaptive reasoning*—capacity for logical thought, reflection, explanation and justification
- *Productive disposition*—habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy

(Kilpatrick, Swafford & Findell, 2001)

These components are different aspects of a complex whole. They are interwoven and interdependent and cannot be easily separated.

How does EQAO's Grade 9 assessment link to *The Ontario Curriculum*?

EQAO's Grade 9 assessment is a curriculum-based, standards-referenced large-scale assessment. It is developed in relation to the *Ontario Curriculum* expectations and standards (level of achievement) for students performance.

The Grade 9 assessment recognized the two Grade 9 courses, academic and applied, and addresses them with different versions of the assessment. The differences between the applied and the academic curricula dictate these two versions. The descriptors of mathematical content and processes below are found on pages 9, 29 and 38 of *The Ontario Curriculum, Grades 9 and 10: Mathematics* (2005).

Principles of Mathematics (Academic)

Mathematical Content Descriptors

Number Sense and Algebra
• Operating with Exponents
• Manipulating Expressions and Solving Equations

Linear Relations
• Using Data Management to Investigate Relationships
• Understanding Characteristics of Linear Relations
• Connecting Various Representations of Linear Relations

Analytic Geometry
• Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph
• Investigating the Properties of Slope
• Using the Properties of Linear Relations to Solve Problems

Measurement and Geometry
• Investigating the Optimal Values of Measurements
• Solving Problems Involving Perimeter, Area, Surface Area, and Volume
• Investigating and Applying Geometric Relationships

Foundations of Mathematics (Applied)

Mathematical Content Descriptors

Number Sense and Algebra
• Solving Problems Involving Proportional Reasoning
• Simplifying Expressions and Solving Equations

Linear Relations
• Using Data Management to Investigate Relationships
• Determining Characteristics of Linear Relations
• Investigating Constant Rate of Change
• Connecting Various Representations of Linear Relations and Solving Problems Using the Representations

Measurement and Geometry
• Investigating the Optimal Values of Measurements of Rectangles
• Solving Problems Involving Perimeter, Area, and Volume
• Investigating and Applying Geometric Relationships

Principles of Mathematics (Academic) and Foundations of Mathematics (Applied) Mathematical Process Descriptors

These mathematical process expectations from *The Ontario Curriculum* are to be integrated into student learning associated with all the strands.

Throughout this course, students will develop their skills and abilities in the following areas:

Problem Solving: develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding.

Reasoning and Proving: develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments.

Reflecting: demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions).

Selecting Tools and Computational Strategies: select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems.

Connecting: make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports).

Representing: create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems.

Communicating: communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Insight:

How the definition of mathematics aligns with current research

Current research in mathematics teaching and learning recognizes that children learn more mathematics when instruction is based on their ways of thinking and engages them in problem solving (Yackel, 1997; Yackel & Cobb, 1996; Zack & Graves, 2001). Students also appear to benefit from teachers assisting them in seeing the connections among various mathematical ideas (Boaler, 2002). Hence, mathematical concepts are not just transmitted but are the result of questioning, probing, making mistakes, reflecting and reworking.

This is an active process in which the student plays a central role in trying to make sense of his or her experiences. These processes of constructing new learning can happen more easily and effectively if the students are working in a rich learning environment.

The Grade 9 assessment offers students opportunities to demonstrate a broad range of mathematical processes in the content strands. This helps to honour the focus on developing understanding through meaningful mathematics activity in the classroom.

What is assessed?

Students are required to demonstrate both content expectations and cognitive processes. The assessment focuses on key aspects of mathematics across the strands in the Grade 9 mathematics curriculum:

- Number Sense and Algebra
- Linear Relations
- Analytic Geometry (academic only)
- Measurement and Geometry

Further, the assessment allows students to demonstrate that they can

- understand concepts;
- apply procedures;
- apply and adapt a variety of appropriate strategies to solve problems;
- use concrete materials to model mathematical ideas;
- make and investigate mathematical conjectures;
- select and use a variety of types of reasoning;
- communicate their mathematical thinking coherently;
- analyze the mathematical thinking of others;
- use appropriate mathematical language and conventions;
- connect mathematical ideas;
- recognize and apply mathematics in a variety of contexts;
- create and use representations to organize, record and communicate mathematically and
- use representations to model mathematical thinking.

(National Council of Teachers of Mathematics, 2000)

CHAPTER 4: The Assessment Process

In This Chapter

- What does the assessment consist of?
- How does EQAO ensure that English language learners and students with special education needs can participate fairly?

Insight: Understanding Ontario's student achievement levels

What does the assessment consist of?

The Grade 9 Assessment of Mathematics consists of two booklets to be written in two sessions. *Administering the Grade 9 Assessment of Mathematics: A Guide For Teachers and Principals* allots 50 minutes per session for student work.

The booklets contain 24 multiple-choice and seven open-response questions that are operational. The questions are distributed among the academic strands (Number Sense and Algebra; Linear Relations; Analytic Geometry; and Measurement and Geometry) and the applied strands (Number Sense and Algebra; Linear Relations; and Measurement and Geometry). (See Chapter 5 for details of the distribution of items across the strands and expectations in the revised, 2005 mathematics curriculum.)

Operational items are "live" test questions that count toward the student's score. Field-test questions are embedded in the test for trial purposes for potential use in future tests. Each booklet contains embedded field-test questions that account for less than 20% of the allotted time.

Grade 9 Assessment of Mathematics: Approximate Number of Items by Type			
	Multiple-Choice Items	Open-Response Items	Total Items
Operational	24	7	31
Field Test	3	1	4
Total Items for Each Student	27	8	35

Grade 9 Assessment of Mathematics: Approximate Number of Raw Score Points and Percentage of Total Raw Score Points by Item Type		
Operational Item Type	Number of Raw Score Points	Percentage of Total Raw Score Points
Multiple-Choice	24	46%
Open-Response	28	54%
Total	52	100%

Note: Only students' responses to the operational items are used to determine their achievement on the assessment.

How does EQAO ensure that English language learners and students with special education needs can participate fairly?

English language learners are provided with special provisions and students with special education needs are allowed accommodations to ensure that they can participate in the Grade 9 Assessment of Mathematics and can demonstrate the full extent of their skills. Each year, EQAO reviews and updates these provisions and accommodations to ensure that they reflect new developments in supports for students. A separate document for students with special education needs and English language learners outlines the policies and procedures for granting accommodations and special provisions, ensuring the integrity of the assessment.

Insight:

Understanding Ontario's student achievement levels

EQAO uses the definitions of the Ontario Ministry of Education levels of achievement for the levels used on its assessments:

> **Level 1** identifies achievement that falls much below the provincial standard, while still reflecting a passing grade.
>
> **Level 2** identifies achievement that approaches the standard.
>
> **Level 3** represents the provincial standard of achievement.
>
> **Level 4** identifies achievement that surpasses the standard.
>
> The characteristics given for Level 3 in the achievement charts in *The Ontario Curriculum* correspond to the provincial standard for achievement of the curriculum expectations. Parents of students achieving at Level 3 can be confident that their children will be prepared for work in the next grade.
>
> It should be noted that achievement at Level 4 does not mean that the student has achieved expectations beyond those specified for a particular course. It indicates that the student has achieved all or almost all of the expectations for that course, and that he or she demonstrates the ability to use the knowledge and skills specified for that course in more sophisticated ways than a student achieving at Level 3. (*The Ontario Curriculum, Grades 9–12: Program Planning and Assessment* [2000])

After all items in a student's performance are scored, the data from the operational items are used to determine the student's level of performance. The Individual Student Report shows both the level and the range within the level at which the student performed. This enables parents and teachers to plan for improvement.

CHAPTER 5: Curriculum Connections and the Blueprint

In This Chapter

• How are curriculum expectations reflected in the assessment?

How are curriculum expectations reflected in the assessment?

The Grade 9 assessment blueprint in this chapter presents the expectations in clusters and gives the number and types of items on the assessment.

Some expectations cannot be appropriately assessed within the limits of a large-scale paper-and-pencil assessment. For instance, it is difficult to assess expectations that require students to perform an investigation. EQAO assessments can assess the knowledge gained through and from an investigation but not how they are designed or carried out. These expectations are best assessed by the teacher in the classroom. For a comparison of large-scale and classroom assessment, see the chart in Chapter 1.

Principles of Mathematics (Academic)

In Grade 9 academic mathematics, some expectations and parts of other expectations cannot be assessed on a large-scale assessment. In the blueprint on the following pages, the expectations or parts thereof that cannot be measured appropriately by a large-scale assessment appear in italics. The blueprint sets the test design for 2006–2007 and subsequent years.

Grade 9 Academic Blueprint

	Grade 9 Academic Mathematics Expectations		
	Academic Mathematical Process Expectations Although the Grade 9 assessment does not measure the process expectations, students are required to apply the following mathematical processes in order to demonstrate success on the assessment. **Problem Solving** • develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding; **Reasoning and Proving** • develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments; **Reflecting** • demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions); **Selecting Tools and Computational Strategies** • select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems; **Connecting** • make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports); **Representing** • create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems; **Communicating** • communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.		

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
	Number Sense and Algebra		
NAV.01	**Number Sense and Algebra, Overall Expectation 1** demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions	2 MC	2 x 1 = 2 score points or 4% of total score
	Number Sense and Algebra, Specific Expectations for Overall 1: Operating with Exponents		
NA1.01	substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $(\frac{3}{2})^3$ by hand and 9.8^3 by using a calculator]) (*Sample problem:* A movie theatre wants to compare the volumes of popcorn in two containers, a cube with edge length 8.1 cm and a cylinder with radius 4.5 cm and height 8.0 cm. Which container holds more popcorn?)		
NA1.02	describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by $(x)(x)$ or x^2; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or x^3]		
NA1.03	*derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials,* and apply these rules in expressions involving one and two variables with positive exponents		
NA1.04	*extend the multiplication rule to derive* and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents		
NAV.02	**Number Sense and Algebra, Overall Expectation 2** manipulate numerical and polynomial expressions, and solve first-degree equations	3 MC 1 OR	3 x 1 + 1 x 4 = 7 score points or 13% of total score
	Number Sense and Algebra, Specific Expectations for Overall 2: Manipulating Expressions and Solving Equations		
NA2.01	simplify numerical expressions involving integers and rational numbers, *with and without the use of technology*		
NA2.02	solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion		
NA2.03	relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations		
NA2.04	add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], *using a variety of tools* (e.g., algebra tiles, computer algebra systems, paper and pencil)		
NA2.05	multiply a polynomial by a monomial involving the same variable [e.g., $2x(x + 4)$, $2x^2(3x^2 - 2x + 1)$], *using a variety of tools* (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil)		
NA2.06	expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], *using a variety of tools* (e.g., algebra tiles, computer algebra systems, paper and pencil)		
NA2.07	solve first-degree equations, including equations with fractional coefficients, *using a variety of tools* (e.g., computer algebra systems, paper and pencil) *and strategies* (e.g., the balance analogy, algebraic strategies)		
NA2.08	rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement) (*Sample problem:* A circular garden has a circumference of 30 m. What is the length of a straight path that goes through the centre of this garden?)		
NA2.09	solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods (*Sample problem:* Solve the following problem in more than one way: Jonah is involved in a walkathon. His goal is to walk 25 km. He begins at 9:00 a.m. and walks at a steady rate of 4 km/h. How many kilometres does he still have left to walk at 1:15 p.m. if he is to achieve his goal?)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
	Linear Relations		
LRV.01	**Linear Relations, Overall Expectation 1** apply data-management techniques to investigate relationships between two variables	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
	Linear Relations, Specific Expectations for Overall 1: Using Data Management to Investigate Relationships		
LR1.01	interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant [e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student] (*Sample problem:* Given a graph that represents the relationship of the Celsius scale and the Fahrenheit scale, determine the Celsius equivalent of –5 °F.)		
LR1.02	*pose problems, identify variables, and formulate hypotheses associated with relationships between two variables (Sample problem: Does the rebound height of a ball depend on the height from which it was dropped?)*		
LR1.03	*design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs) (Sample problem: Design and perform an experiment to measure and record the temperature of ice water in a plastic cup and ice water in a thermal mug over a 30 min period, for the purpose of comparison. What factors might affect the outcome of this experiment? How could you design the experiment to account for them?)*		
LR1.04	describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?) (*Sample problem:* Hypothesize the effect of the length of a pendulum on the time required for the pendulum to make five full swings. Use data to make an inference. Compare the inference with the hypothesis. Are there other relationships you might investigate involving pendulums?)		
LRV.02	**Linear Relations, Overall Expectation 2** demonstrate an understanding of the characteristics of a linear relation		
	Linear Relations, Specific Expectations for Overall 2: Understanding Characteristics of Linear Relations		
LR2.01	construct tables of values, graphs, and equations, *using a variety of tools* (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations (*Sample problem:* Construct a table of values, a graph, and an equation to represent a monthly cellphone plan that costs $25, plus $0.10 per minute of airtime.)		
LR2.02	construct tables of values, scatter plots, and lines or curves of best fit as appropriate, *using a variety of tools* (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials) (*Sample problem:* Collect data, using concrete materials or dynamic geometry software, and construct a table of values, a scatter plot, and a line or curve of best fit to represent the following relationships: the volume and the height for a square-based prism with a fixed base; the volume and the side length of the base for a square-based prism with a fixed height.)		
LR2.03	identify, *through investigation*, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear		
LR2.04	compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation) (*Sample problem:* Yoga costs $20 for registration, plus $8 per class. Tai chi costs $12 per class. Which situation represents a direct variation, and which represents a partial variation? For each relation, what is the initial value? Explain your answers.)		
LR2.05	determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
LRV.03	**Linear Relations, Overall Expectation 3** connect various representations of a linear relation	2 MC 1 OR	2 x 1 + 1 x 4 = 6 score points or 12% of total score
	Linear Relations, Specific Expectations for Overall 3: Connecting Various Representations of Linear Relations		
LR3.01	determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation (*Sample problem*: The equation $H = 300 - 60t$ represents the height of a balloon that is initially at 300 m and is descending at a constant rate of 60 m/min. Determine algebraically and graphically how long the balloon will take to reach a height of 160 m.)		
LR3.02	describe a situation that would explain the events illustrated by a given graph of a relationship between two variables (*Sample problem:* The walk of an individual is illustrated in the given graph, produced by a motion detector and a graphing calculator. Describe the walk [e.g., the initial distance from the motion detector, the rate of walk].)		
LR3.03	determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model)		
LR3.04	describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation)		

MC multiple-choice item OR open-response item

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
	Analytic Geometry		
AGV.01	**Analytic Geometry, Overall Expectation 1** determine the relationship between the form of an equation and the shape of its graph with respect to linearity and non-linearity	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
	Analytic Geometry, Specific Expectations for Overall 1: Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph		
AG1.01	determine, *through investigation*, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation)		
AG1.02	identify, *through investigation*, the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$		
AG1.03	express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$		
AGV.02	**Analytic Geometry, Overall Expectation 2** determine, *through investigation*, the properties of the slope and y-intercept of a linear relation		
	Analytic Geometry, Specific Expectations for Overall 2: Investigating the Properties of Slope		
AG2.01	determine, *through investigation*, various formulas for the slope of a line segment or a line; (e.g., $m = \frac{rise}{run}$, $m = \frac{\text{the change in } y}{\text{the change in } x}$ or $m = \frac{\Delta y}{\Delta x}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$), and use the formulas to determine the slope of a line segment or a line		
AG2.02	identify, *through investigation with technology,* the geometric significance of m and b in the equation $y = mx + b$		
AG2.03	determine, *through investigation*, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is $50, plus $5 per book, so an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation)		
AG2.04	identify, *through investigation*, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate		

MC multiple-choice item   OR open-response item

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
AGV.03	**Analytic Geometry, Overall Expectation 3** solve problems involving linear relations	3 MC 1 OR	3 x 1 + 1 x 4 = 7 score points or 13% of total score
	Analytic Geometry, Specific Expectations for Overall 3: Using the Properties of Linear Relations to Solve Problems		
AG3.01	graph lines by hand, using a variety of techniques (e.g., graph $y = \frac{2}{3}x - 4$ using the *y*-intercept and slope; graph $2x + 3y = 6$ using the *x*- and *y*-intercepts)		
AG3.02	determine the equation of a line from information about the line (e.g., the slope and *y*-intercept; the slope and a point; two points) (*Sample problem*: Compare the equations of the lines parallel to and perpendicular to $y = 2x - 4$, and with the same *x*-intercept as $3x - 4y = 12$. Verify using dynamic geometry software.)		
AG3.03	describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package)		
AG3.04	identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to $50 to $2550)		
AG3.05	determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application (*Sample problem*: A video rental company has two monthly plans. Plan A charges a flat fee of $30 for unlimited rentals; Plan B charges $9, plus $3 per video. Use a graphical model to determine the conditions under which you should choose Plan A or Plan B.)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
	Measurement and Geometry		
MGV.01	**Measurement and Geometry, Overall Expectation 1** determine, *through investigation*, the optimal values of various measurements	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
	Measurement and Geometry, Specific Expectations for Overall 1: Investigating the Optimal Values of Measurements		
MG1.01	determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, *using a variety of tools* (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant		
MG1.02	determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, *using a variety of tools* (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant		
MG1.03	identify, *through investigation with a variety of tools* (e.g., concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area]		
MG1.04	explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss)		
MG1.05	pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides) (*Sample problem*: Determine the dimensions of a square-based, open-topped prism with a volume of 24 cm^3 and with the minimum surface area.)		
MGV.02	**Measurement and Geometry, Overall Expectation 2** solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures		
	Measurement and Geometry, Specific Expectations for Overall 2: Solving Problems Involving Perimeter, Area, Surface Area, and Volume		
MG2.01	relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$		
MG2.02	solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone)		
MG2.03	solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles) (*Sample problem*: A new park is in the shape of an isosceles trapezoid with a square attached to the shortest side. The side lengths of the trapezoidal section are 200 m, 500 m, 500 m, and 800 m, and the side length of the square section is 200 m. If the park is to be fully fenced and sodded, how much fencing and sod are required?)		
MG2.04	develop, *through investigation* (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is $\frac{1}{3}$ the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{pyramid} = \frac{V_{prism}}{3}$ or $V_{pyramid} = \frac{(\text{area of base})(\text{height})}{3}$)		
MG2.05	determine, *through investigation*, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square-based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles)		
MG2.06	solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures (*Sample problem*: Break-bit Cereal is sold in a single-serving size, in a box in the shape of a rectangular prism of dimensions 5 cm by 4 cm by 10 cm. The manufacturer also sells the cereal in a larger size, in a box with dimensions double those of the smaller box. Compare the surface areas and the volumes of the two boxes, and explain the implications of your answers.)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Academic Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
MGV.03	**Measurement and Geometry, Overall Expectation 3** verify, *through investigation facilitated by dynamic geometry software*, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems	2 MC 1 OR	2 x 1 + 1 x 4 = 6 score points or 12% of total score
	Measurement and Geometry, Specific Expectations for Overall 3: Investigating and Applying Geometric Relationships		
MG3.01	determine, *through investigation using a variety of tools* (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons (*Sample problem*: With the assistance of dynamic geometry software, determine the relationship between the sum of the interior angles of a polygon and the number of sides. Use your conclusion to determine the sum of the interior angles of a 20-sided polygon.)		
MG3.02	determine, *through investigation using a variety of tools* (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides)		
MG3.03	*pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables) (Sample problem: How many diagonals can be drawn from one vertex of a 20-sided polygon? How can I find out without counting them?)*		
MG3.04	*illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software (Sample problem: Confirm or deny the following statement: If a quadrilateral has perpendicular diagonals, then it is a square.)*		
	MC multiple-choice item OR open-response item	**24 MC** **7 OR**	**Total Raw Score Points = 52 or 100%* of total score**

*Because percentages in the blueprint are rounded, they may not add up to 100.

Foundations of Mathematics (Applied)

In Grade 9 applied mathematics, some expectations and parts of other expectations cannot be assessed on a large-scale assessment. In the blueprints on the following pages, expectations or parts thereof that cannot be measured appropriately by a large-scale assessment appear in italics. The blueprint sets the test design for 2006–2007 and subsequent years.

Grade 9 Applied Blueprint

	Grade 9 Applied Mathematics Expectations		
	Applied Mathematical Process Expectations Although the Grade 9 assessment does not measure the process expectations, students are required to apply the mathematical processes in order to demonstrate success on the assessment. **Problem Solving** • develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding; **Reasoning and Proving** • develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments; **Reflecting** • demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions); **Selecting Tools and Computational Strategies** • select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems; **Connecting** • make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports); **Representing** • create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems; **Communicating** • communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.		

Number	Grade 9 Applied Mathematics Expectations	MC Total = 24 OR Total = 7	Raw Score Points
	Number Sense and Algebra		
NAV.01	**Number Sense and Algebra, Overall Expectation 1** solve problems involving proportional reasoning	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
	Number Sense and Algebra, Specific Expectations for Overall 1: Solving Problems Involving Proportional Reasoning		
NA1.01	illustrate equivalent ratios, *using a variety of tools* (e.g., concrete materials, diagrams, dynamic geometry software) (e.g., show that 4:6 represents the same ratio as 2:3 by showing that a ramp with a height of 4 m and a base of 6 m and a ramp with a height of 2 m and a base of 3 m are equally steep)		
NA1.02	represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations (*Sample problem*: You are building a skateboard ramp whose ratio of height to base must be 2:3. Write a proportion that could be used to determine the base if the height is 4.5 m.)		
NA1.03	solve for the unknown value in a proportion, using a variety of methods (e.g., concrete materials, algebraic reasoning, equivalent ratios, constant of proportionality) (*Sample problem*: Solve $\frac{x}{4} = \frac{15}{20}$.)		
NA1.04	make comparisons using unit rates (e.g., if 500 mL of juice costs $2.29, the unit rate is 0.458¢/mL; this unit rate is less than for 750 mL of juice at $3.59, which has a unit rate of 0.479¢/mL)		
NA1.05	solve problems involving ratios, rates, and directly proportional relationships in various contexts (e.g., currency conversions, scale drawings, measurement), using a variety of methods (e.g., using algebraic reasoning, equivalent ratios, a constant of proportionality; using dynamic geometry software to construct and measure scale drawings) (*Sample problem*: Simple interest is directly proportional to the amount invested. If Luis invests $84 for one year and earns $1.26 in interest, how much would he earn in interest if he invested $235 for one year?)		
NA1.06	solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms (e.g., calculating simple interest and sales tax; analysing data) (*Sample problem*: Of the 29 students in a Grade 9 math class, 13 are taking science this semester. If this class is representative of all the Grade 9 students in the school, estimate and calculate the percent of the 236 Grade 9 students who are taking science this semester. Estimate and calculate the number of Grade 9 students this percent represents.)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Applied Mathematics Expectations	MC Total = 24 OR Total = 7	Raw Score Points
NAV.02	**Number Sense and Algebra, Overall Expectation 2** simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations	3 MC 1 OR	3 x 1 + 1 x 4 = 7 score points or 13% of total score
	Number Sense and Algebra, Specific Expectations for Overall 2: Simplifying Expressions and Solving Equations		
NA2.01	simplify numerical expressions involving integers and rational numbers, *with and without the use of technology*		
NA2.02	relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations		
NA2.03	describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by $(x)(x)$ or x^2; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or x^3]		
NA2.04	substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $(\frac{3}{7})^3$ by hand and 9.8^3 by using a calculator]) (*Sample problem*: A movie theatre wants to compare the volumes of popcorn in two containers, a cube with edge length 8.1 cm and a cylinder with radius 4.5 cm and height 8.0 cm. Which container holds more popcorn?)		
NA2.05	add and subtract polynomials involving the same variable up to degree three [e.g., $(2x + 1) + (x^2 - 3x + 4)$], *using a variety of tools* (e.g., algebra tiles, computer algebra systems, paper and pencil)		
NA2.06	multiply a polynomial by a monomial involving the same variable to give results up to degree three [e.g., $(2x)(3x)$, $2x(x + 3)$], *using a variety of tools* (e.g., algebra tiles, drawings, computer algebra systems, paper and pencil)		
NA2.07	solve first-degree equations with non-fractional coefficients, *using a variety of tools* (e.g., computer algebra systems, paper and pencil) *and strategies* (e.g., the balance analogy, algebraic strategies) (Sample problem: Solve $2x + 7 = 6x - 1$ using the balance analogy.)		
NA2.08	substitute into algebraic equations and solve for one variable in the first degree (e.g., in relationships, in measurement) (*Sample problem*: The perimeter of a rectangle can be represented as $P = 2l + 2w$. If the perimeter of a rectangle is 59 cm and the width is 12 cm, determine the length.)		

MC multiple-choice item OR open-response item

Number	Grade 9 Applied Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
	Linear Relations		
LRV.01	**Linear Relations, Overall Expectation 1** apply data-management techniques to investigate relationships between two variables		
	Linear Relations, Specific Expectations for Overall 1: **Using Data Management to Investigate Relationships**		
LR1.01	interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant [e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student] (*Sample problem*: Given a graph that represents the relationship of the Celsius scale and the Fahrenheit scale, determine the Celsius equivalent of –5 °F.)		
LR1.02	*pose problems, identify variables, and formulate hypotheses associated with relationships between two variables (Sample problem: Does the rebound height of a ball depend on the height from which it was dropped?)*		
LR1.03	*carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs) (Sample problem: Perform an experiment to measure and record the temperature of ice water in a plastic cup and ice water in a thermal mug over a 30 min period, for the purpose of comparison. What factors might affect the outcome of this experiment? How could you change the experiment to account for them?)*		
LR1.04	describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?) (*Sample problem*: Hypothesize the effect of the length of a pendulum on the time required for the pendulum to make five full swings. Use data to make an inference. Compare the inference with the hypothesis. Are there other relationships you might investigate involving pendulums?)	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
LRV.02	**Linear Relations, Overall Expectation 2** determine the characteristics of linear relations		
	Linear Relations, Specific Expectations for Overall 2: **Determining Characteristics of Linear Relations**		
LR2.01	construct tables of values and graphs, *using a variety of tools* (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations (*Sample problem*: Construct a table of values and a graph to represent a monthly cellphone plan that costs $25, plus $0.10 per minute of airtime.)		
LR2.02	construct tables of values, scatter plots, and lines or curves of best fit as appropriate, *using a variety of tools* (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials) (*Sample problem*: Collect data, using concrete materials or dynamic geometry software, and construct a table of values, a scatter plot, and a line or curve of best fit to represent the following relationships: the volume and the height for a square-based prism with a fixed base; the volume and the side length of the base for a square-based prism with a fixed height.)		
LR2.03	identify, *through investigation*, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear		

MC multiple-choice item   OR open-response item

Number	Grade 9 Applied Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
LRV.03	**Linear Relations, Overall Expectation 3** demonstrate an understanding of constant rate of change and its connection to linear relations		
	Linear Relations, Specific Expectations for Overall 3: Investigating Constant Rate of Change		
LR3.01	determine, *through investigation*, that the rate of change of a linear relation can be found by choosing any two points on the line that represents the relation, finding the vertical change between the points (i.e., the rise) and the horizontal change between the points (i.e., the run), and writing the ratio $\frac{rise}{run}$ (i.e., *rate of change* = $\frac{rise}{run}$)		
LR3.02	determine, *through investigation*, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is $50, plus $5 per book, so an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5; and 5 is the coefficient of the independent variable, p, in this equation)	3 MC 1 OR	3 x 1 + 1 x 4 = 7 score points or 13% of total score
LR3.03	compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation) (*Sample problem:* Yoga costs $20 for registration, plus $8 per class. Tai chi costs $12 per class. Which situation represents a direct variation, and which represents a partial variation? For each relation, what is the initial value? Explain your answers.)		
LR3.04	express a linear relation as an equation in two variables, using the rate of change and the initial value (e.g., Mei is raising funds in a charity walkathon; the course measures 25 km, and Mei walks at a steady pace of 4 km/h; the distance she has left to walk can be expressed as $d = 25 - 4t$, where t is the number of hours since she started the walk)		
LR3.05	describe the meaning of the rate of change and the initial value for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Applied Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
LRV.04	**Linear Relations, Overall Expectation 4** connect various representations of a linear relation, and solve problems using the representations	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
	Linear Relations, Specific Expectations for Overall 4: Connecting Various Representations of Linear Relations and Solving Problems Using the Representations		
LR4.01	determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation (*Sample problem:* The equation $H = 300 - 60t$ represents the height of a hot air balloon that is initially at 300 m and is descending at a constant rate of 60 m/min. Determine algebraically and graphically its height after 3.5 min.)		
LR4.02	describe a situation that would explain the events illustrated by a given graph of a relationship between two variables (*Sample problem:* The walk of an individual is illustrated in the given graph, produced by a motion detector and a graphing calculator. Describe the walk [e.g., the initial distance from the motion detector, the rate of walk].)		
LR4.03	determine other representations of a linear relation arising from a realistic situation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model)		
LR4.04	solve problems that can be modelled with first-degree equations, and compare the algebraic method to other solution methods (e.g., graphing) (*Sample problem:* Bill noticed it snowing and measured that 5 cm of snow had already fallen. During the next hour, an additional 1.5 cm of snow fell. If it continues to snow at this rate, how many more hours will it take until a total of 12.5 cm of snow has accumulated?)		
LR4.05	describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation)		
LR4.06	determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application (*Sample problem:* A video rental company has two monthly plans. Plan A charges a flat fee of $30 for unlimited rentals; Plan B charges $9, plus $3 per video. Use a graphical model to determine the conditions under which you should choose Plan A or Plan B.)		
LR4.07	*select a topic involving a two-variable relationship (e.g., the amount of your pay cheque and the number of hours you work; trends in sports salaries over time; the time required to cool a cup of coffee), pose a question on the topic, collect data to answer the question, and present its solution using appropriate representations of the data (Sample problem: Individually or in a small group, collect data on the cost compared to the capacity of computer hard drives. Present the data numerically, graphically, and [if linear] algebraically. Describe the results and any trends orally or by making a poster display or by using presentation software.)*		

MC multiple-choice item   OR open-response item

Number	Grade 9 Applied Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
	Measurement and Geometry		
MGV.01	**Measurement and Geometry, Overall Expectation 1** determine, *through investigation*, the optimal values of various measurements of rectangles	4 MC 1 OR	4 x 1 + 1 x 4 = 8 score points or 15% of total score
	Measurement and Geometry, Specific Expectations for Overall 1: Investigating the Optimal Values of Measurements of Rectangles		
MG1.01	determine the maximum area of a rectangle with a given perimeter *by constructing a variety of rectangles, using a variety of tools* (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant		
MG1.02	determine the minimum perimeter of a rectangle with a given area *by constructing a variety of rectangles, using a variety of tools* (e.g., geoboards, graph paper, a pre-made dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant		
MG1.03	solve problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area (*Sample problem:* You have 100 m of fence to enclose a rectangular area to be used for a snow sculpture competition. One side of the area is bounded by the school, so the fence is required for only three sides of the rectangle. Determine the dimensions of the maximum area that can be enclosed.)		
MGV.02	**Measurement and Geometry, Overall Expectation 2** solve problems involving the measurements of two-dimensional shapes and the volumes of three-dimensional figures		
	Measurement and Geometry, Specific Expectations for Overall 2: Solving Problems Involving Perimeter, Area, and Volume		
MG2.01	relate the geometric representation of the Pythagorean theorem to the algebraic representation $a^2 + b^2 = c^2$		
MG2.02	solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone)		
MG2.03	solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles) (*Sample problem:* A new park is in the shape of an isosceles trapezoid with a square attached to the shortest side. The side lengths of the trapezoidal section are 200 m, 500 m, 500 m, and 800 m, and the side length of the square section is 200 m. If the park is to be fully fenced and sodded, how much fencing and sod are required?)		
MG2.04	develop, *through investigation* (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is $\frac{1}{3}$ the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{pyramid} = \frac{V_{prism}}{3}$ or $V_{pyramid} = \frac{(\text{area of base})(\text{height})}{3}$)		
MG2.05	solve problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres (*Sample problem:* Break-bit Cereal is sold in a single-serving size, in a box in the shape of a rectangular prism of dimensions 5 cm by 4 cm by 10 cm. The manufacturer also sells the cereal in a larger size, in a box with dimensions double those of the smaller box. Make a hypothesis about the effect on the volume of doubling the dimensions. Test your hypothesis using the volumes of the two boxes, and discuss the result.)		

MC multiple-choice item   OR open-response item

Number	Grade 9 Applied Mathematics Expectations	Item Types MC Total = 24 OR Total = 7	Raw Score Points
MGV.03	**Measurement and Geometry, Overall Expectation 3** determine, *through investigation facilitated by dynamic geometry software,* geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems	2 MC 1 OR	2 x 1 + 1 x 4 = 6 score points or 12% of total score
	Measurement and Geometry, Specific Expectations for Overall 3: Investigating and Applying Geometric Relationships		
MG3.01	determine, *through investigation using a variety of tools* (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons (*Sample problem:* With the assistance of dynamic geometry software, determine the relationship between the sum of the interior angles of a polygon and the number of sides. Use your conclusion to determine the sum of the interior angles of a 20-sided polygon.)		
MG3.02	determine, *through investigation using a variety of tools* (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the angles formed by parallel lines cut by a transversal, and apply the results to problems involving parallel lines (e.g., given a diagram of a rectangular gate with a supporting diagonal beam, and given the measure of one angle in the diagram, use the angle properties of triangles and parallel lines to determine the measures of the other angles in the diagram)		
MG3.03	*create an original dynamic sketch, paper folding design, or other illustration that incorporates some of the geometric properties from this section, or find and report on some real-life application(s) (e.g., in carpentry, sports, architecture) of the geometric properties*		
	MC multiple-choice item OR open-response item	**24** MC **7** OR	**Total Raw Score Points = 52 or 100%* of total score**

*Because percentages in the blueprint are rounded, they may not add up to 100.

CHAPTER 6: How the Assessment Is Scored

In This Chapter

• How is the assessment scored?

How is the assessment scored?

Each open-response item on the Grade 9 assessment is scored according to a guide called an "item-specific rubric." On this page is the general (or "generic") rubric from which the item-specific rubrics are developed. Multiple-choice items are scored by machine.

Generic Grade 9 Assessment Rubric for Open-Response Mathematics Questions

Code	Descriptor
Blank	• blank: nothing written or drawn in response to the question
Illegible/ Off topic	• illegible: cannot be read; completely crossed out/erased; not written in English • irrelevant content: does not attempt assigned question (e.g., comment on the task, drawings, "?", "!", "I don't know") • off topic: no relationship of written work to the question

Code 10	• demonstration of limited understanding of concepts and/or procedures
	• application of knowledge and skills shows limited effectiveness due to
	  • misunderstanding of concepts
	  • incorrect selection or misuse of procedures
	• problem-solving process shows limited effectiveness due to
	  • minimal evidence of a solution process
	  • limited identification of important elements of the problem
	  • too much emphasis on unimportant elements of the problem
	  • no conclusions presented
	  • conclusion presented without supporting evidence

Code 20	• demonstration of some understanding of concepts and/or procedures
	• application of knowledge and skills shows some effectiveness due to
	  • partial understanding of the concepts
	  • errors and/or omissions in the application of the procedures
	• problem-solving process shows some effectiveness due to
	  • an incomplete solution process
	  • identification of some of the important elements of the problem
	  • some understanding of the relationships between important elements of the problem
	  • simple conclusions with little supporting evidence

Code 30	• demonstration of considerable understanding of concepts and/or procedures
	• application of knowledge and skills shows considerable effectiveness due to
	  • an understanding of most of the concepts
	  • minor errors and/or omissions in the application of the procedures
	• problem-solving process shows considerable effectiveness due to
	  • a solution process that is nearly complete
	  • identification of most of the important elements of the problem
	  • a considerable understanding of the relationships between important elements of the problem
	  • appropriate conclusions with supporting evidence

Code 40	• demonstration of thorough understanding of concepts and/or procedures
	• application of knowledge and skills shows a high degree of effectiveness due to
	  • a thorough understanding of the concepts
	  • an accurate application of the procedures (any minor errors and/or omissions do not detract from the demonstration of a thorough understanding)
	• problem-solving process shows a high degree of effectiveness due to
	  • a complete solution process
	  • identification of all important elements of the problem
	  • a thorough understanding of the relationships between all of the important elements of the problem
	  • appropriate conclusions with thorough and insightful supporting evidence

CHAPTER 7: Maintaining Comparability

In This Chapter

- How is the comparability of the assessment maintained from year to year?
- How is the assessment blueprint used?
- How are the assessments equated year to year?
- Why and how are items field tested?

How is the comparability of the assessment maintained from year to year?

It is critically important that EQAO assessments be comparable from year to year. A number of measures are taken to ensure year-to-year comparability, including

- use of an assessment blueprint;
- equating assessments from year to year and
- use of field-test items.

How is the assessment blueprint used?

EQAO has developed a blueprint so that the assessment has the same characteristics each year. The blueprint presents the expectations from *The Ontario Curriculum* in clusters. The blueprint gives the number of multiple-choice and open-response questions on the assessment that measure each cluster of expectations.

Although questions on the Grade 9 assessment are allocated to clusters of expectations as indicated in Chapter 5, they are developed to address a specific expectation in the cluster. From year to year, different specific expectations are addressed. Chapter 5 identifies expectations that cannot be assessed appropriately on large-scale assessments and consequently will never have questions mapped to them.

How are the assessments equated year to year?

Data on field tested items are used in the construction of each new version of the assessment so the assessment in a given year is similar in difficulty to that in the previous year. EQAO employs statistical equating procedures that place student scores in two adjacent years on a common scale. This controls for small differences in difficulty from year to year and ensures that the data at the school, board and provincial levels can be validly compared from year to year. The student skills and knowledge required to demonstrate achievement at a given level are consistent from year to year.

Why and how are items field tested?

Embedded field-test items are used to try out new assessment items before they become operational to ensure they are fair for all students and to equate the assessment with those of previous years, so that results can be compared from one year to the next.

Field-test items look like the operational part of the booklet. However, scores on field-test items are not used in determining student, school, board or provincial results.

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